Aula 4a – Filtragem espacial

Conceitos, correlação e convolução

Prof. João Fernando Mari

joaof.mari@ufv.br

Correlação e convolução

f(x,y)

	0	1	2	3
0	f(-1,-1)	f(-1, 0)	f(-1, 1)	...
1	f(0, -1)	f(0, 0)	f(0, 1)	...
2	f(1, -1)	f(1, 0)	f(1, 1)	...
3	...	...	...	...

w(s,t)

	-1	0	1
-1	w(-1,-1)	w(-1, 0)	w(-1, 1)
0	w(0, -1)	w(0, 0)	w(0, 1)
1	w(1, -1)	w(1, 0)	w(1, 1)

Correlação

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

Convolução

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x-s, y-t)$$

Correlação e convolução

f(x,y)

	0	1	2	3
0	***f(0, 0)***	***f(0, 1)***	...	...
1	***f(1, 0)***	***f(1, 1)***	...	...
2	...	...	...	...
3	...	...	...	...

w(s,t)

	-1	0	1
-1	***w(-1,-1)***	***w(-1, 0)***	***w(-1, 1)***
0	***w(0, -1)***	***w(0, 0)***	***w(0, 1)***
1	***w(1, -1)***	***w(1, 0)***	***w(1, 1)***

padding!

Correlação

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

Convolução

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x-s, y-t)$$

Convolução

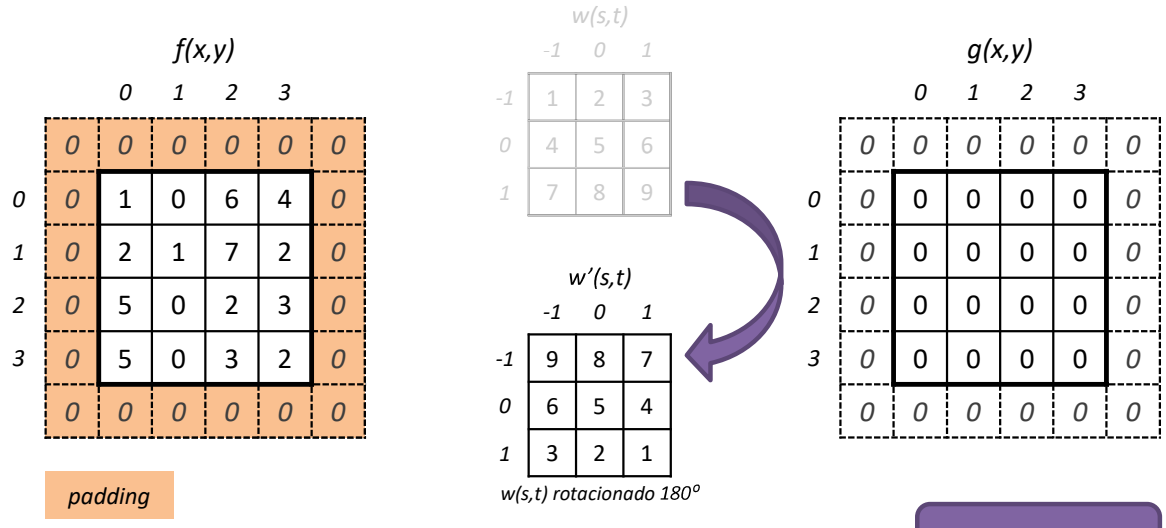
f(x,y)

	0	1	2	3
0	1	0	6	4
1	2	1	7	2
2	5	0	2	3
3	5	0	3	2

w(s,t)

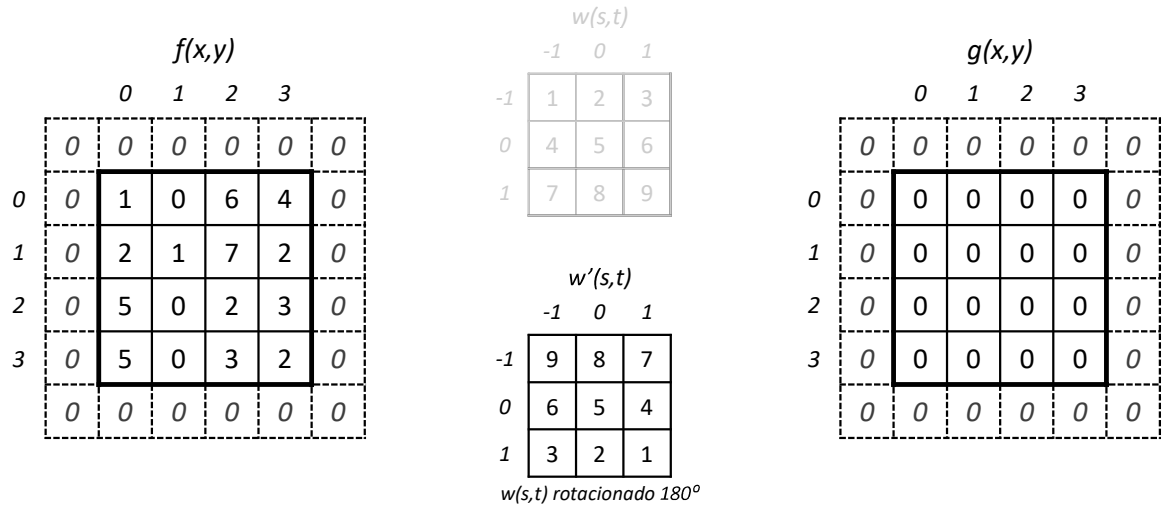
	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

Convolução



$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

Convolução



$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

Convolução

f(x,y)

	0	1	2	3	
	0	0	0	0	0	0
0	0	1	0	6	4	0
1	0	2	1	7	2	0
2	0	5	0	2	3	0
3	0	5	0	3	2	0
	0	0	0	0	0	0

w(s,t)

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

w'(s,t)

	-1	0	1
-1	9	8	7
0	6	5	4
1	3	2	1

w(s,t) rotacionado 180°

g(x,y)

	0	1	2	3	
	0	0	0	0	0	0
0	0	10	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
	0	0	0	0	0	0

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) = \begin{matrix} 9\times 0 + 8\times 0 + 7\times 0 + \\ 6\times 0 + 5\times 1 + 4\times 0 + \\ 3\times 0 + 2\times 2 + 1\times 1 \end{matrix} = 10$$

Convolução

f(x,y)

	0	1	2	3	
	0	0	0	0	0	0
0	0	1	0	6	4	0
1	0	2	1	7	2	0
2	0	5	0	2	3	0
3	0	5	0	3	2	0
	0	0	0	0	0	0

w(s,t)

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

w'(s,t)

	-1	0	1
-1	9	8	7
0	6	5	4
1	3	2	1

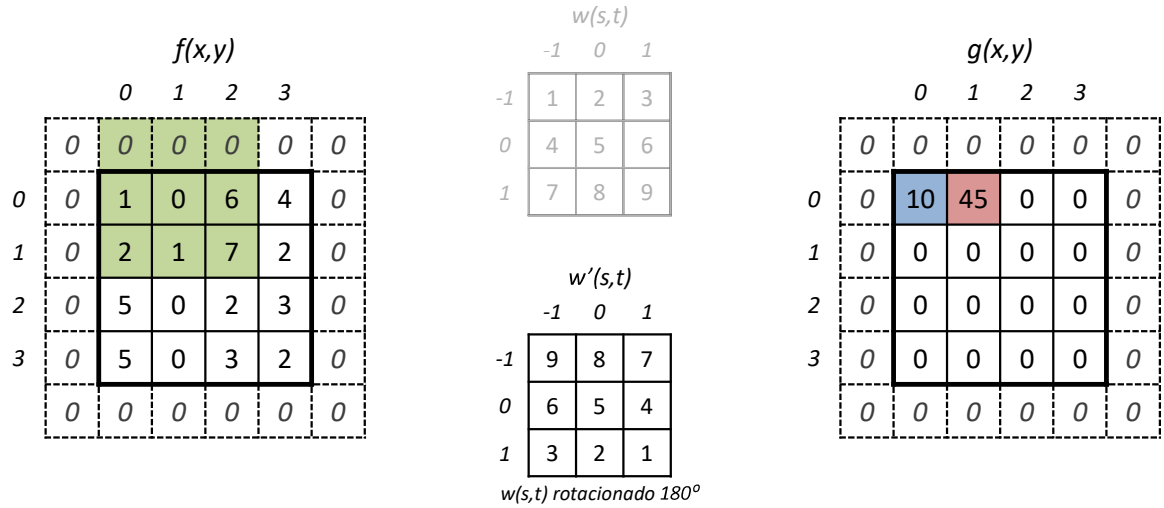
w(s,t) rotacionado 180°

g(x,y)

	0	1	2	3	
	0	0	0	0	0	0
0	0	10	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
	0	0	0	0	0	0

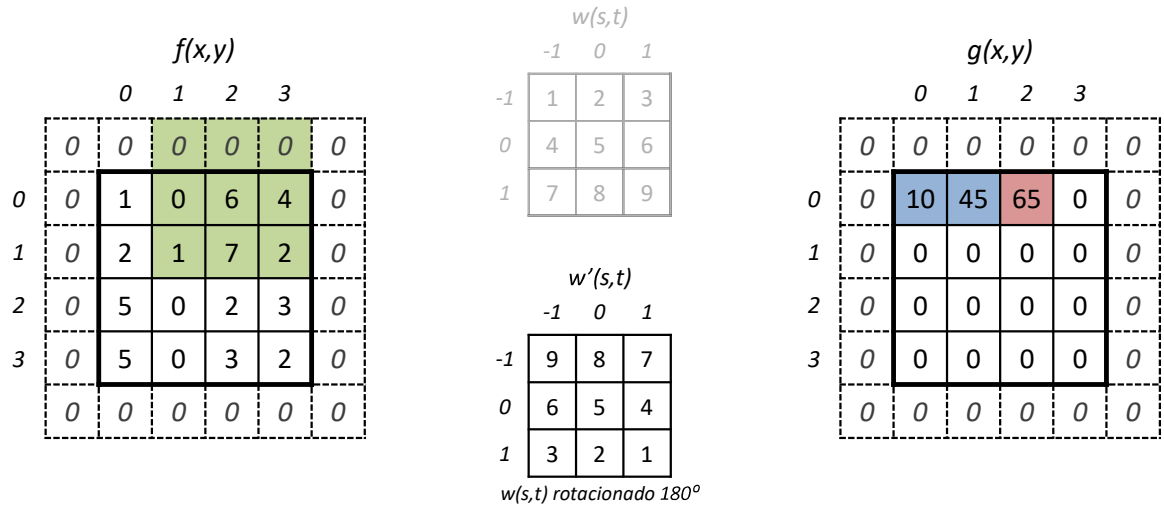
$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) = \begin{matrix} 9\times 0 + 8\times 0 + 7\times 0 + \\ 6\times 0 + 5\times 1 + 4\times 0 + \\ 3\times 0 + 2\times 2 + 1\times 1 \end{matrix} = 10$$

Convolução



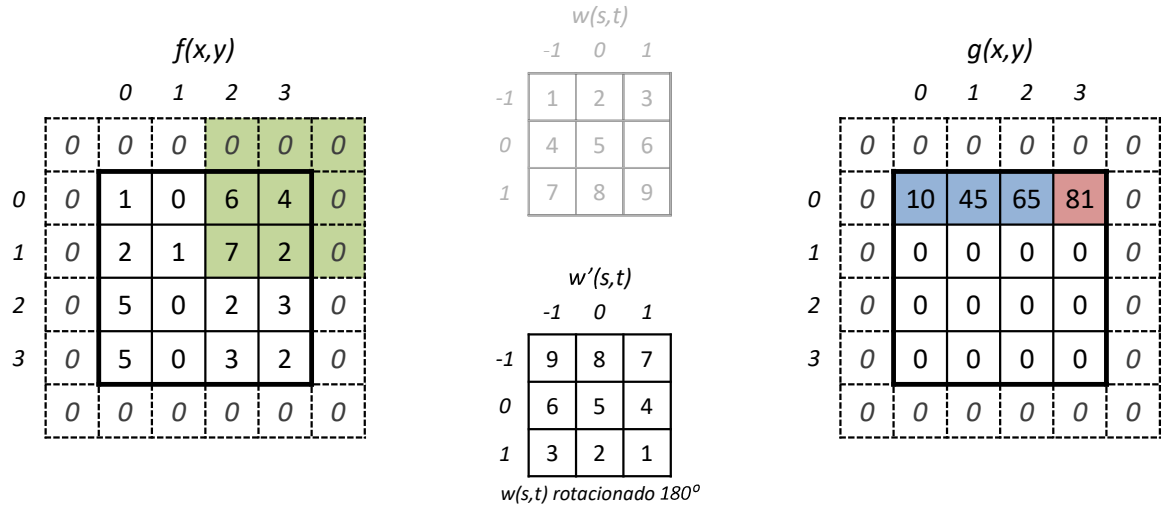
$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) = \begin{matrix} 9\times 0 + 8\times 0 + 7\times 0 + \\ 6\times 1 + 5\times 0 + 4\times 6 + \\ 3\times 2 + 2\times 1 + 1\times 7 \end{matrix} = 45$$

Convolução



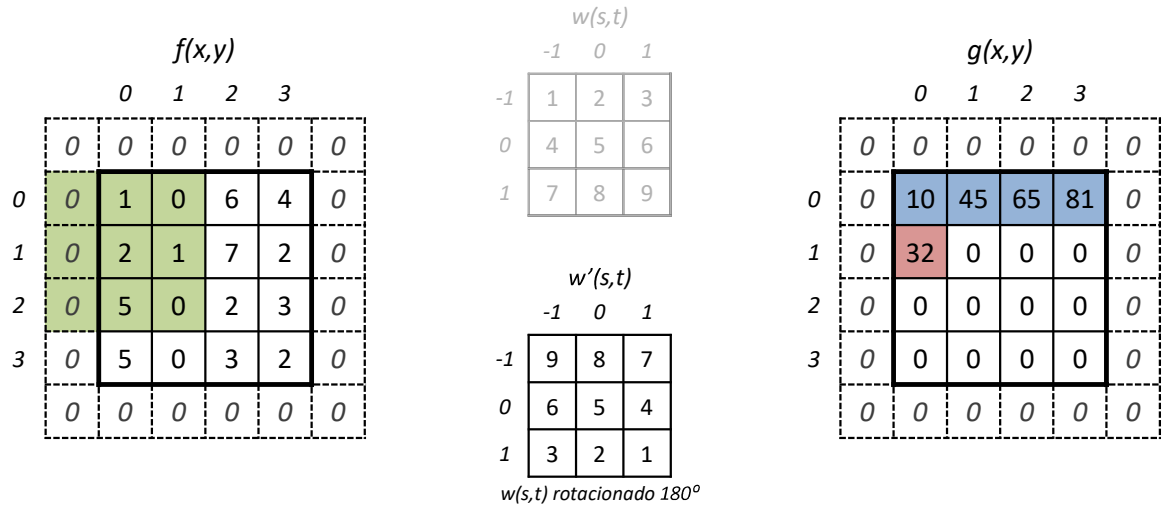
$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) = \begin{matrix} 0\times 9 + 0\times 8 + 0\times 7 \\ 0\times 6 + 6\times 5 + 4\times 4 \\ 1\times 3 + 7\times 2 + 2\times 1 \end{matrix} = 65$$

Convolução



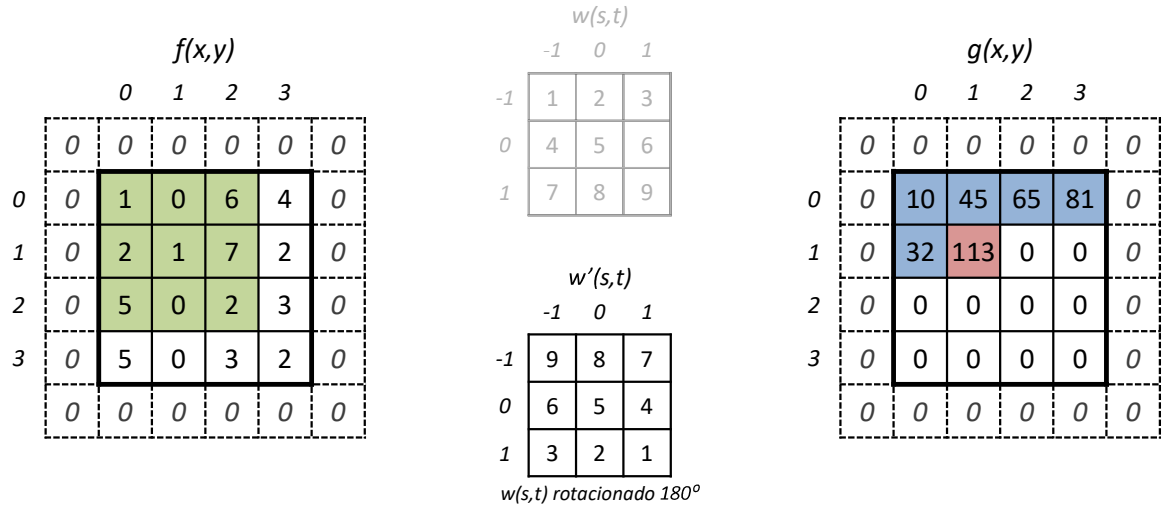
$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) = \begin{matrix} 0\times 9 + 0\times 8 + 0\times 7 \\ 6\times 6 + 4\times 5 + 0\times 4 \\ 7\times 3 + 2\times 2 + 0\times 1 \end{matrix} = 81$$

Convolução



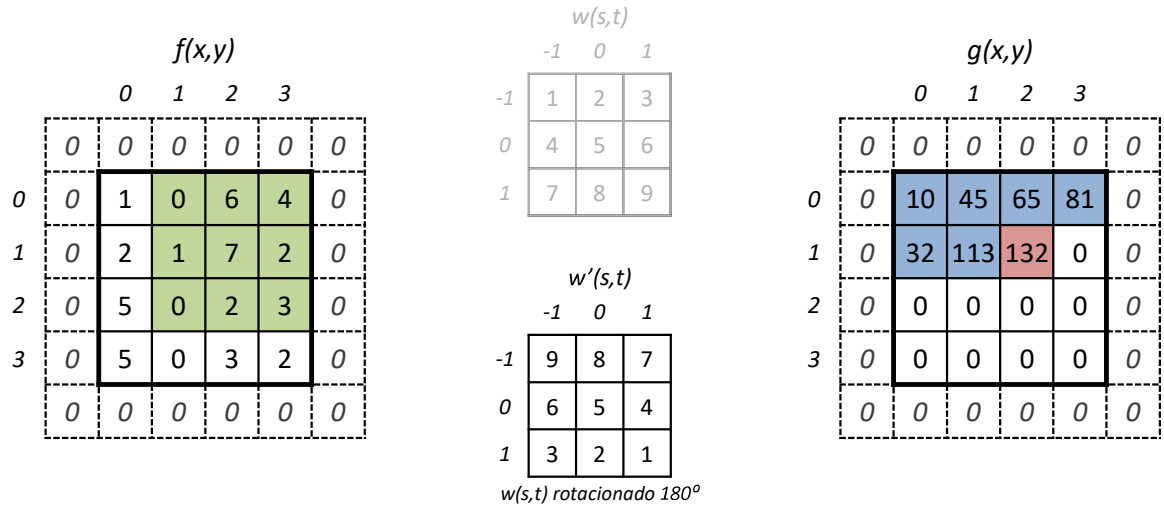
$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) = \begin{matrix} 0\times 9 + 1\times 8 + 0\times 7 \\ 0\times 6 + 2\times 5 + 1\times 4 \\ 0\times 3 + 5\times 2 + 0\times 1 \end{matrix} = 32$$

Convolução



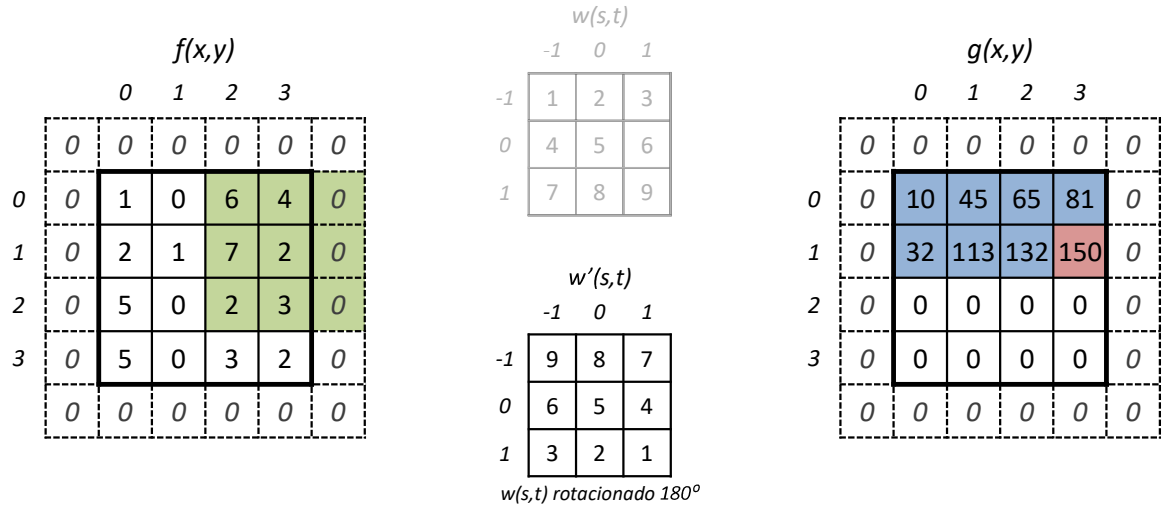
$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) = \begin{matrix} 1\times 9 + 0\times 8 + 6\times 7 \\ 2\times 6 + 1\times 5 + 7\times 4 \\ 5\times 3 + 0\times 2 + 2\times 1 \end{matrix} = 113$$

Convolução



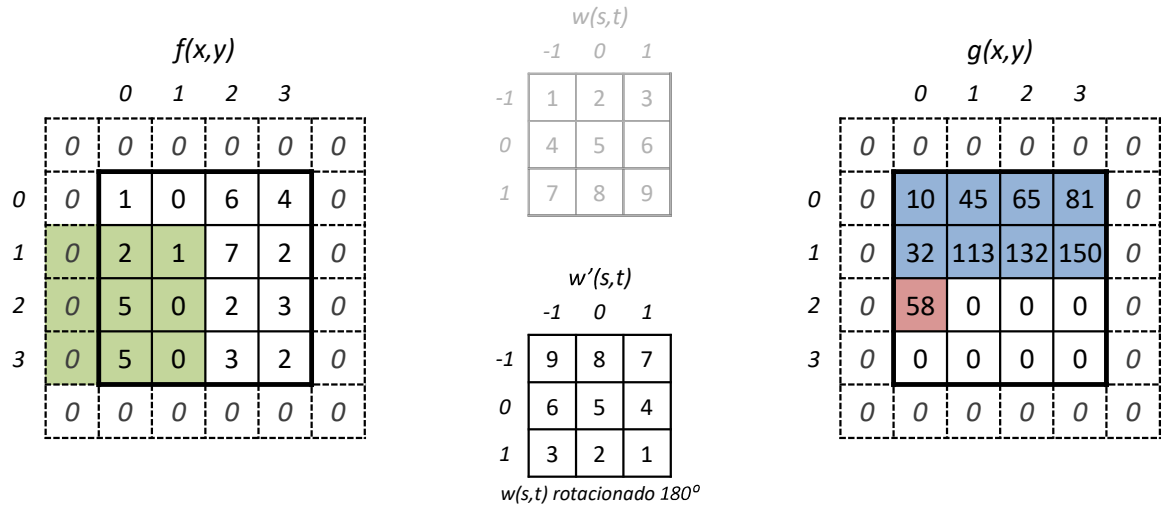
$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) = \begin{matrix} 0\times 9 + 6\times 8 + 4\times 7 \\ 1\times 6 + 7\times 5 + 2\times 4 \\ 0\times 3 + 2\times 2 + 3\times 1 \end{matrix} = 132$$

Convolução



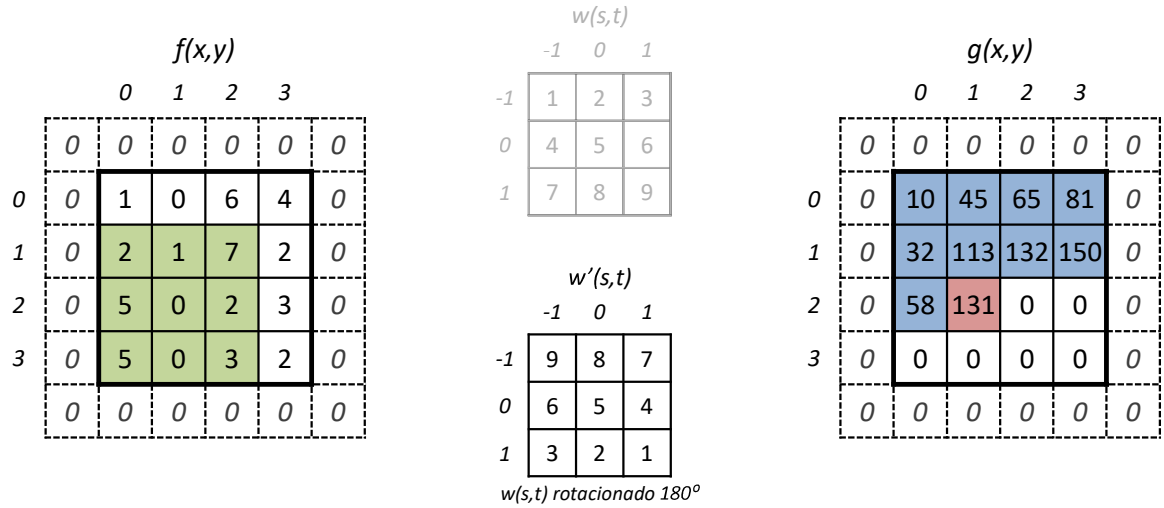
$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) = \begin{matrix} 6\times 9 + 4\times 8 + 0\times 7 \\ 7\times 6 + 2\times 5 + 0\times 4 \\ 2\times 3 + 3\times 2 + 0\times 1 \end{matrix} = 150$$

Convolução



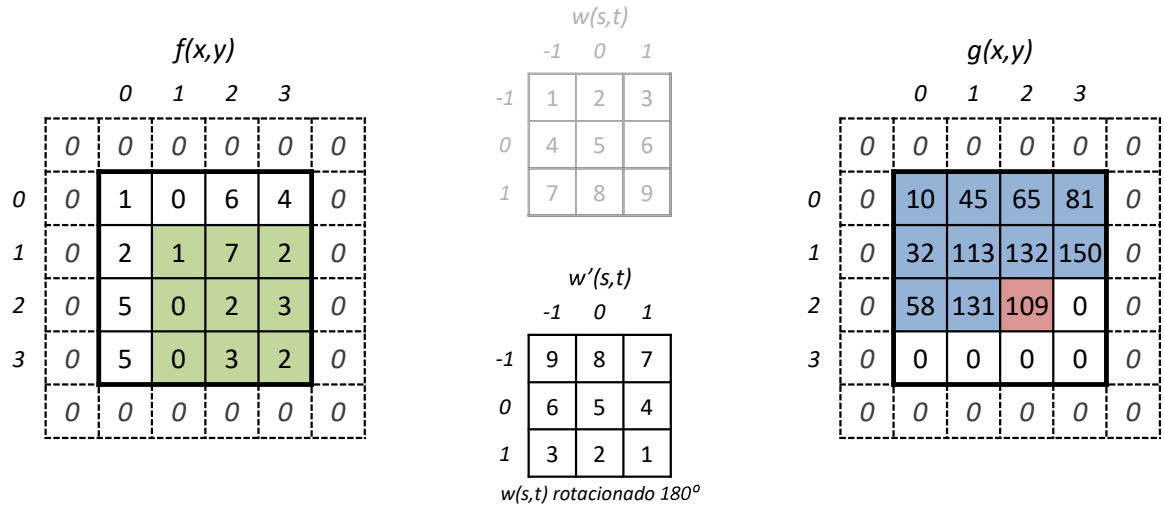
$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) = \begin{matrix} 0\times 9 + 2\times 8 + 1\times 7 \\ 0\times 6 + 5\times 5 + 0\times 4 \\ 0\times 3 + 5\times 2 + 0\times 1 \end{matrix} = 58$$

Convolução



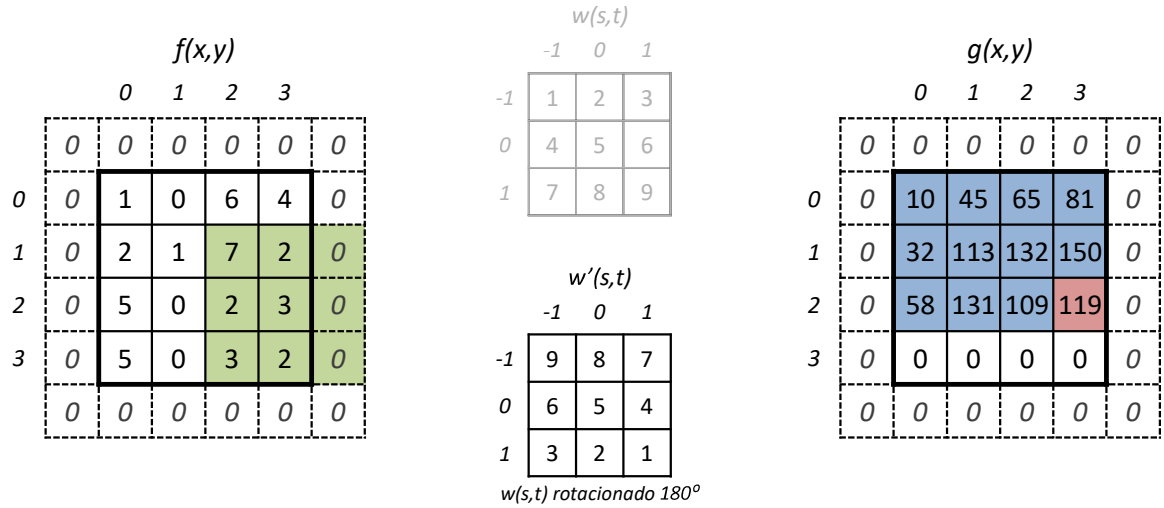
$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) = \begin{matrix} 2\times 9 + 1\times 8 + 7\times 7 \\ 5\times 6 + 0\times 5 + 2\times 4 \\ 5\times 3 + 0\times 2 + 3\times 1 \end{matrix} = 131$$

Convolução



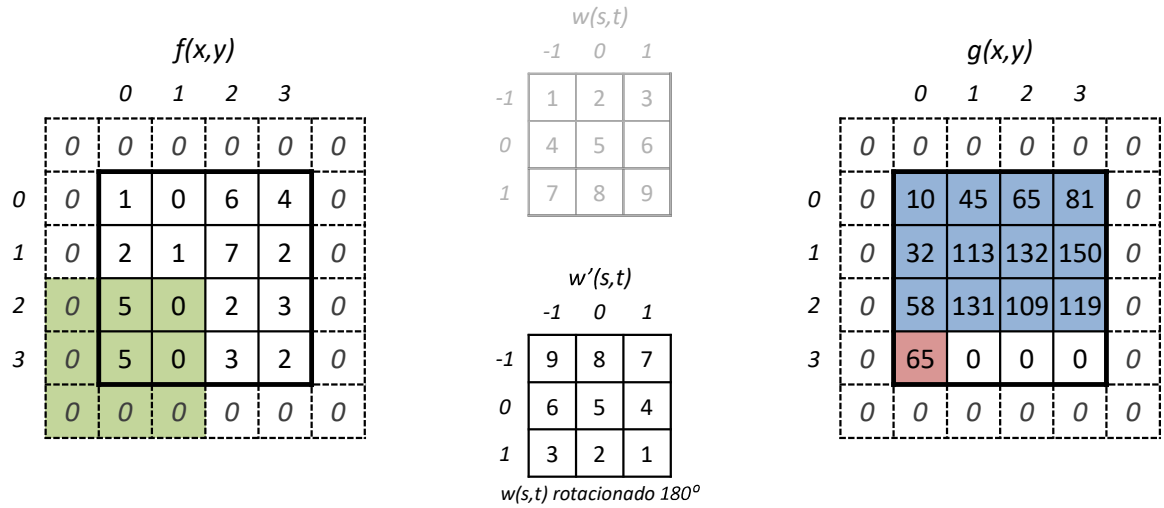
$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) = \begin{matrix} 1\times 9 + 7\times 8 + 2\times 7 \\ 0\times 6 + 2\times 5 + 3\times 4 \\ 0\times 3 + 3\times 2 + 2\times 1 \end{matrix} = 109$$

Convolução



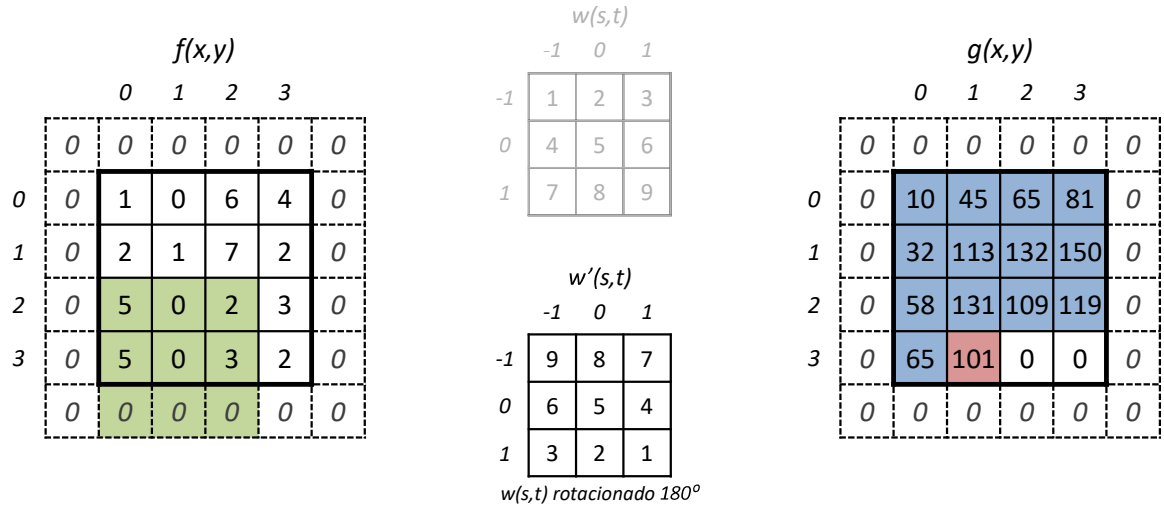
$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) = \begin{matrix} 7\times 9 + 2\times 8 + 0\times 7 \\ 2\times 6 + 3\times 5 + 0\times 4 \\ 3\times 3 + 2\times 2 + 0\times 1 \end{matrix} = 119$$

Convolução



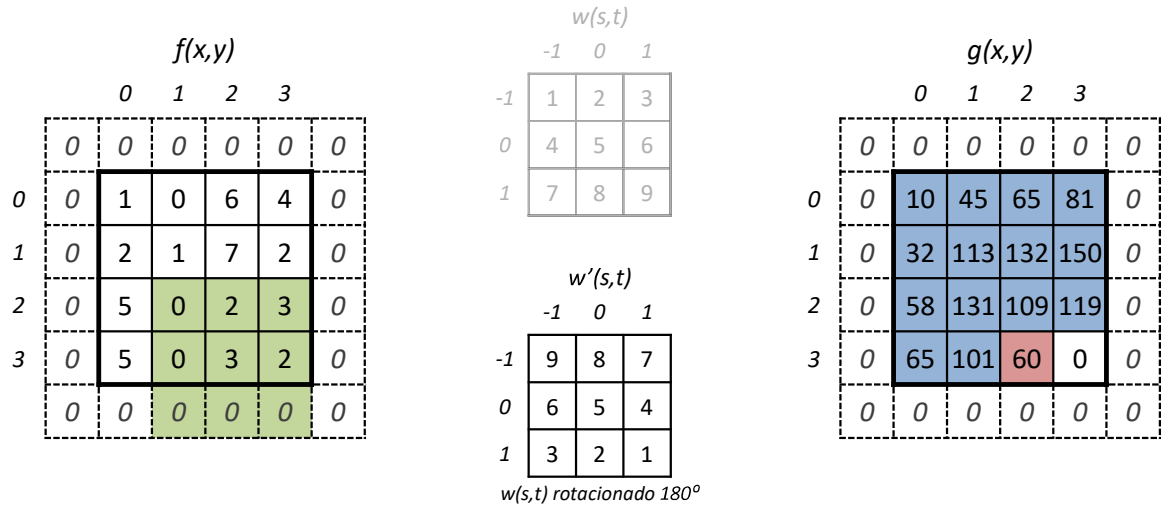
$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) = \begin{matrix} 0\times 9 + 5\times 8 + 0\times 7 \\ 0\times 6 + 5\times 5 + 0\times 4 \\ 0\times 3 + 0\times 2 + 0\times 1 \end{matrix} = 65$$

Convolução



$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) = \begin{matrix} 5\times 9 + 0\times 8 + 2\times 7 \\ 5\times 6 + 0\times 5 + 3\times 4 \\ 0\times 3 + 0\times 2 + 0\times 1 \end{matrix} = 101$$

Convolução



$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) = \begin{matrix} 0\times 9 + 2\times 8 + 3\times 7 \\ 0\times 6 + 3\times 5 + 2\times 4 \\ 0\times 3 + 0\times 2 + 0\times 1 \end{matrix} = 60$$

Convolução

f(x,y)

	0	1	2	3	
	0	0	0	0	0
0	0	1	0	6	4
1	0	2	1	7	2
2	0	5	0	2	3
3	0	5	0	3	2
	0	0	0	0	0

w(s,t)

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

w'(s,t)

	-1	0	1
-1	9	8	7
0	6	5	4
1	3	2	1

w(s,t) rotacionado 180°

g(x,y)

	0	1	2	3	
	0	0	0	0	0
0	0	10	45	65	81
1	0	32	113	132	150
2	0	58	131	109	119
3	0	65	101	60	70
	0	0	0	0	0

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) = \begin{matrix} 2\times 9 + 3\times 8 + 0\times 7 \\ 3\times 6 + 2\times 5 + 0\times 4 \\ 0\times 3 + 0\times 2 + 0\times 1 \end{matrix} = 70$$

Convolução

f(x,y)

	0	1	2	3	
	0	0	0	0	0
0	0	1	0	6	4
1	0	2	1	7	2
2	0	5	0	2	3
3	0	5	0	3	2
	0	0	0	0	0

w(s,t)

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

w'(s,t)

	-1	0	1
-1	9	8	7
0	6	5	4
1	3	2	1

w(s,t) rotacionado 180°

g(x,y)

	0	1	2	3	
	0	0	0	0	0
0	0	10	45	65	81
1	0	32	113	132	150
2	0	58	131	109	119
3	0	65	101	60	70
	0	0	0	0	0

Convolução

f(x,y)

	0	1	2	3
0	1	0	6	4
1	2	1	7	2
2	5	0	2	3
3	5	0	3	2

w(s,t)

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

w'(s,t)

	-1	0	1
-1	9	8	7
0	6	5	4
1	3	2	1

w(s,t) rotacionado 180°

g(x,y)

	0	1	2	3
0	10	45	65	81
1	32	113	132	150
2	58	131	109	119
3	65	101	60	70

Bibliografia

MARQUES FILHO, O.; VIEIRA NETO, H. **Processamento digital de imagens**. Brasport, 1999.

Disponível para download no site do autor (Exclusivo para uso pessoal)

http://dainf.ct.utfpr.edu.br/~hvieir/pub.html

GONZALEZ, R.C.; WOODS, R.E.; **Processamento Digital de Imagens.** 3ª edição. Editora Pearson, 2009.

Disponível na Biblioteca Virtual da Pearson.

J. E. R. Queiroz, H. M. Gomes. **Introdução ao Processamento Digital de Imagens**. RITA. v. 13, 2006.

http://www.dsc.ufcg.edu.br/~hmg/disciplinas/graduacao/vc-2016.2/Rita-Tutorial-PDI.pdf

FIM